

# Numerical schemes for rate-independent phase-field damage models

For rate-independent phase-field damage models several different numerically and analytically motivated solution schemes have been designed. A common strategy in the engineering community is alternate minimization (AM) [BFM], which exploits the separately convex structure of the underlying optimization problem.

From a mathematical point of view we face the difficulty that solutions might show a discontinuous evolution. In this regard several, in general not equivalent, solution concepts have been developed. Within this talk the focus lies on balanced viscosity (BV) solutions. In [BRKM] a new scheme to approximate BV solutions of a damage model [BFM] was developed. It combines AM with local minimization and a convergence proof is provided.

Most application related numerical examples suggest that solutions generated by the pure AM scheme converge to BV solutions as well. A first convergence analysis for the pure AM scheme was carried out in [KN] for the damage model. However, it still did not answer the question, if AM solutions are indeed BV solutions. The characterizations of the two types of solutions were similar but not identical.

To answer the question and justify the new scheme with its additional computational cost, we present in the first part of this talk some finite dimensional examples where the pure AM scheme does not generate BV solutions.

In the second part of this talk, the focus is on the implementation of the scheme from [BRKM] within a FEM framework. A robust and efficient numerical solver is presented, combining AM with an augmented Lagrangian approach. The algorithm is carefully analysed from a physics point of view. Particularly, the implementation and effect of different norms and arclength increments in the framework [BRKM] are investigated and the obtained solution is compared to a purely AM generated solution.

[BRKM] S. Boddin, F. Rörentrop, D. Knees, J. Mosler. Approximation of balanced viscosity solutions of a rate-independent damage model by combining alternate minimization with a local minimization algorithm. *arXiv:2211.12940*, 2022.

[KN] D. Knees, M. Negri. Convergence of alternate minimization schemes for phase field fracture and damage. *Mathematical Models and Methods in Applied Sciences (M3AS)*, vol. 27(9), pp. 1743-1794, 2017.

[BFM] B. Bourdin, G.A. Francfort, J.-J. Marigo. Numerical experiments in revisited brittle fracture. *Journal of the Mechanics and Physics of Solids*, vol. 48(4), pp. 797-826, 2000.