Thursday, September 23rd, 14:00 – 15:00

Dr. Flaviana Iurlano

Title: Phase-field approximation of cohesive fracture energies

Abstract: Variational models in Fracture Mechanics are effectively described through functional spaces with discontinuities. The most renowned example is Griffith's energy for brittle fracture, describing a situation in which already for the smallest opening there is no interaction between the two sides of the crack. In ductile materials, crack proceeds rather through the opening of a series of voids separated by thin filaments, which produce a weak bond between the lips at moderate openings (cohesive fracture).

A large literature has been devoted to the derivation of models including interfaces from more regular models, like damage or phase-field models, mainly within the framework of Gamma-convergence. These approximations can be interpreted both as microscopic physical models, so that the Gamma-convergence justifies the macroscopic model, and as regularization, therefore they can be used for example in numerical simulations. The first work of this sort is Ambrosio--Tortorelli '90, which provides a phase-field approximation of the Mumford-Shah functional for image segmentation. The corresponding result in the case of antiplane shear mode (scalar-valued displacements) cohesive fracture has been only recently obtained in Conti--Focardi--Iurlano '16.

The general case of vector-valued displacements, in a geometrically nonlinear framework, is the object of a work in preparation with S. Conti and M. Focardi. In our phase-field models the elastic coefficient is computed from the damage variable v through the function $f_\varepsilon(v):=\min\{1,\varepsilon^{\frac{1}{2}} f(v)\}$, with f diverging for v close to the value describing undamaged material. The resulting absolute continuous density is quasiconvex with 1-growth at infinity, while the fracture density, depending on the opening of the crack and on the normal vector to the crack set, is given in terms of an asymptotic n-dimensional formula, is bounded, and has a linear behavior for small openings.

Thursday, September 23rd, 15:30 – 16:30

Prof. Dr. Laura De Lorenzis

Three new contributions to phase-field modeling of brittle fracture

Laura De Lorenzis, Tymofiy Gerasimov, Corrado Maurini, Ulrich Römer, Jaroslav Vondrejc, Hermann Matthies

The phase-field modeling approach to fracture has recently attracted a great deal of attention due to its remarkable capability to naturally handle fracture phenomena with arbitrarily complex crack topologies in three dimensions. On one side, the approach can be obtained through the regularization of the variational approach to fracture introduced by Francfort and Marigo in 1998, which is conceptually related to Griffith's view of fracture; on the other side, it can be constructed as a gradient damage model with some specific properties. The functional to be minimized is not convex, so that the necessary stationarity conditions of the functional may admit multiple solutions. The solution obtained in an actual computation is typically one out of several local minimizers.

In this talk, the speaker highlights three recent contributions to phase-field modeling of brittle fracture. In the first part of the talk, the focus is placed on the issue of multiple solutions. Here a paradigm shift is advocated, away from the search for one particular solution towards the simultaneous description of all possible solutions (local minimizers), along with the probabilities of their occurrence. We propose the stochastic relaxation of the variational brittle fracture problem through random perturbations of the functional and introduce the concept of stochastic solution represented by random fields. In the numerical experiments, we use a simple Monte Carlo approach to compute approximations to such stochastic solutions. The final result of the computation is not a single crack pattern, but rather several possible crack patterns and their probabilities. The stochastic solution framework using evolving random fields allows additionally the interesting possibility of conditioning the probabilities of further crack paths on intermediate crack patterns.

The second part of the talk focuses on crack nucleation under multiaxial stress states. It is shown that the available energy decompositions, introduced to avoid crack interpenetration and to allow for asymmetric fracture behavior in tension and compression, lead to multiaxial strength surfaces of different but fixed shapes. Thus, once the intrinsic length scale of the phase-field model is tailored to recover the experimental tensile strength, it is not possible to match the experimental compressive or shear strength. The talk introduces a newly proposed energy decomposition that enables the straightforward calibration of a multi-axial failure surface of the Drucker-Prager type. The new decomposition, preserving the variational structure of the model, includes an additional free parameter that can be calibrated based on the experimental ratio of the compressive to the tensile strength (or, if possible, of the shear to the tensile strength), as successfully demonstrated on two data sets taken from the literature.

The third part of the talk deals with brittle fracture in anisotropic materials featuring two-fold and four-fold symmetric fracture toughness. For these two classes, the talk introduces two newly proposed variational phase-field models based on the family of regularizations proposed by Focardi, for which Gamma-convergence results hold. Since both models are of second order, as opposed to the previously available fourth-order models for four-fold symmetric fracture toughness, they do not require basis functions of C1-continuity nor mixed variational principles for finite element discretization. For the four-fold symmetric formulation we show that the standard quadratic degradation function is unsuitable and devise a procedure to derive a suitable one. The performance of the new models is assessed via several numerical examples that simulate anisotropic fracture under anti-plane shear loading.